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Abstract
The development rate of any country's economy is in proportion to the rate of energy consumption. According to energy saving factor, a country is divided in main four sectors: Industrial, Residential, Transportation and Agricultural. In India, Residential sector consumes twenty two percentage of energy from the total energy consumption rate. In Residential sector, Air conditioning system and Refrigeration system consume maximum energy. In the present work, a mathematical model is prepared to demonstrate energy saving potential of different thermal insulating materials by taking air conditioned, insulated room as a specimen. Energy saving potential of different thermal insulation materials is calculated by a Numerical analysis with boundary conditions like solar radiation as heating load and air condition as cooling load applied at different walls. Moreover, the air condition (AC) – cooling load boundary condition is set to depend upon the inside room temperature. i.e when inside room temperature reaches 26.5 °C, the AC is set “on” - absorbing heat from the room while the room temperature reduces to 24 °C the AC is set “off”. The room is discretised using Finite Volume Method to obtained temperature at different nodes. Simulations are carried out for different conductivities and thickness of insulation materials and concluded that the AC on-off time cycle is different for different insulating materials. On the basis of working hours of AC, the amount of energy saved is calculated and compared for different insulation materials.


1. Introduction
The requirement of energy resources increases with rate of energy consumption. Country’s economy can be developed by using energy resources in efficient way. In any developed country, maximum energy is used in industrial and residential sectors. More than 40% from the total consumption of energy is used in industries in India, but it can be decreased by using efficient machinery. After industrial sector, residential sector is one of the most energy consuming in India. In residential sector, energy can be saved by two alternative methods. (1) Design the residential structure in efficient manner so that can be reduced electricity bill or (2) Use of insulation material on wall surfaces. In residential sector, use of insulation material is the most efficient way of energy conservation. The current energy consumption scenario is the main motivation to find and compare the energy saving potential of different insulation material to solve this essential problem related to energy sources.

Jim [1] has examined two different residential buildings by conducting experiment in different weather conditions of Hong Kong. An electricity consumption of air condition was monitored by precision energy loggers. Energy consumption of air conditioning system can be reduced by insulating material on wall surface was concluded during experiment.

A lot of research work has been done on conjugate heat transfer for a different shaped cavities with different boundary condition and with different fluids. In some
problems, the heat generation sources present in enclosure. A large number of studies have been reported on convective heat transfer in rectangular cavities characteristics of the flow because of its application in the insulation of the buildings [2]. Ha et al [3] carried a numerical study on a cube shaped, steady state, conjugate heat transfer consisting of conduction and natural convection phenomenon. 3-D, steady state non-dimensional governing equations have been used during analysis. Temperature difference generated by heat source develop a flow inside the cavity. The temperature ratio of left and right wall can be calculated from the analysis.

Shinsuke et al [4] carried a study on 3-D ventilated room with hybrid AC. The characteristics of the indoor environment have been examined by the use of CFD (Computational fluid Dynamics) simulation processes under various conditions of incoming outdoor air. The distribution of the age of air was under the strong influence of flow fields. Unsteady governing equations with a wide variety of boundary conditions have been solved by numerical techniques. The cooling load of the mechanical AC system increased with the increase of the outdoor air temperature and the decrease of the incoming outdoor air volume was concluded.

In present time, the numerical method is more preferable due to its flexibility and versatility with the use of advance computer technology. The most common preferred numerical methods are: Finite Difference Method, Finite Volume Method, Finite Element Method etc [1]. In general, to write the continuous functions such as temperature in discrete forms by dividing the domain of interest into large number of grids is the basic principle of numerical method. Due to discretization, set of algebraic equations form the governing partial differential equations [4].

2. MATHEMATICAL MODELLING

2.1 Physical Description

A study has been carried on 2D model of room having an air conditioning system which maintains the temperature at 24°C. The inside surfaces of the walls are thermally insulated. To identify the energy saving potential of different thermal insulation material is the main objective. The project involves study of conjugated heat transfer process, which is the combination of conduction heat transfer process and convective heat transfer process. Three boundary conditions are applied on the problem for the calculation of the temperature at different points inside the thermally insulated room. The first condition is the solar radiation falling on the walls and roof of the room. This condition is used for the calculation of inside wall surface temperature that varies with time and direction of sun. The second one is the no-slip condition at the walls. Velocity of the fluid at the walls (boundary) of the room is zero. The third is the constant wall heat flux dissipated by the AC. When the AC is on adiabatic condition is applied on west wall. Inside the room heat is transferred by mixed convection. Temperature inside the room is calculated by the control volume method and use of governing equations of gas dynamics. When the temperature reaches at a particular limit the AC gets on or off. The main aim was to determine the effect of various insulating material on the On-Off cycle of the air conditioner.

![Fig. 1 Insulated room 2D model](image)

2.2 Governing Equations in Non-dimensional Form

In present work non-dimensional form of governing equations have been solved. Governing equations have been converted into non-dimensional form by dividing the flow variables by relevant quantities. Equations can be written in dimensionless form using scaling constant for
length as \( L \), velocity as \( u_m \), pressure as \( P = \frac{p}{\rho u_m^2} \) and time as \( \frac{t}{T} \).

Continuity equation in non-dimensional form for an incompressible fluid can be written as,

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)
\]

X-Momentum equation in non-dimensional form for an incompressible unsteady fluid [7,8] can be written as,

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + \frac{\partial U}{\partial t} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (2)
\]

Y-Momentum equation in non-dimensional form for an incompressible unsteady fluid can be written as,

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + \frac{\partial V}{\partial t} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Gr}{Re^2} \xi \quad (3)
\]

Energy equation in non-dimensional form for an incompressible unsteady fluid can be written as,

\[
U \frac{\partial \xi}{\partial X} + V \frac{\partial \xi}{\partial Y} + \frac{\partial \xi}{\partial t} = \frac{1}{Re.Pr} \left( \frac{\partial^2 \xi}{\partial X^2} + \frac{\partial^2 \xi}{\partial Y^2} \right) \quad (4)
\]

\( Gr \) and \( Re \) are non-dimensional parameters of governing equation and \( \xi \) is used for non-dimensional temperature.

### 2.3 Calculation for Intensity of Solar Radiation

The solar radiation reaching the earth’s surface can be classified in two types, direct solar radiation and diffuse solar radiation [5,6]. Various parameters which are incorporated in the calculation for solar radiation intensity falling over a roof or wall of building are as follows: Latitude (\( \phi \)), Declination angle (\( d \)), Hour angle (\( \omega \)), Surface azimuth angle (\( \gamma \)), Slope (\( S \)), Angle of incident (\( \theta \)), Direct radiation intensity (\( I_D \)), Diffuse radiation intensity (\( I_D \)) and Beam radiation in direction of sun rays(\( I_n \)). Intensity of radiation is calculated for the calculation of Solar-air temperature (\( T_c \)) temperature is as following:

Direct radiation can be calculated by,

\[
I_D = I_n \cos \theta \text{ (W/m}^2\text{)} \quad (5)
\]

Where \( \cos \theta = \sin \phi (\sin \phi \cos S + \cos \phi \cos \gamma \cos \omega \cos S) \)

\[+ \cos (\cos \phi \cos \phi \sin \omega \sin S) \]

Beam radiation in direction of sun rays can be calculated by,

\[
I_n = A e^{-B \cos \phi} \text{ (W/m}^2\text{)} \quad (6)
\]

Diffuse radiation can be calculated by,

\[
I_d = Cl_n F_{SS} \text{ (W/m}^2\text{)} \quad (8)
\]

In Equation (7) and (8) A, B and C are constants whose values have been determined on a month wise basis. These constants change during the year because of seasonal changes in the dust and water vapour content of the atmosphere and also because of the changing earth-sun distance [9]. The values of A, B and C were initially given by Threlkeld and Jordan[10] and subsequently revised by Iqbal[11]. The revised values are given in Table 1.1.

\[
F_{SS} = \text{Radiation shape factor for a tilted surface with respect to sky = (1 + \cos S)/2}
\]

<table>
<thead>
<tr>
<th>Month</th>
<th>A (W/m²)</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 21</td>
<td>1202</td>
<td>0.141</td>
<td>0.103</td>
</tr>
<tr>
<td>February 21</td>
<td>1187</td>
<td>0.142</td>
<td>0.104</td>
</tr>
</tbody>
</table>
Total radiation can be calculated by,

\[ I = I_D + I_d \text{ (W/m}^2\text{)} \]  \hspace{1cm} (9)

### 2.4 Calculation for Wall Inner Surface Temperature

Various methods are available for calculating the inside surface temperature. Inner wall temperature is a boundary condition for calculation of temperature at different nodes in the room which can be changed with insulation used and atmospheric conditions. Finite difference method has been used for calculating inside wall surface temperature.

\[ h_o (T_e - T_s) = \frac{k}{\Delta x} (T_e - T_i) \]  \hspace{1cm} (10)

Where \( h_o \) = convection coefficient, \( k \) = thermal conductivity of insulated wall, \( T_e \) = Wall outer surface temperature, \( T_e \) = Solar-air temperature, \( T_i \) = Wall inner surface temperature.

In a study of heat transfer through the wall, the effect of ambient temperature (atmospheric temperature) and the intensity of solar radiation need to be taken as a single quantity.

Accordingly, Solar-air temperature can be calculated by,

\[ T_e = T_o + \left( \frac{\alpha I_D + \alpha I_d}{h_o} \right) \]  \hspace{1cm} (11)

Where \( T_o \) = atmospheric temperature, \( \alpha \) = Absorptivity of surface and \( h_o \) = convection heat transfer coefficient at outer surface of wall.

Taking \( T_i = T_o \), we get,

\[ T_i = T_o - h_o \frac{\Delta x}{k} (T_o - T_i) \]  \hspace{1cm} (12)

Finally \( T_i \) is converted into non-dimensional form to solve eq. 4.

### 3. NUMERICAL METHODOLOGY

Finite volume method is used for evaluating partial differential equations in form of algebraic equations. Initially the domain is discretised in finite number of subdomains. In fig.3 node P is the computational node covered from the north, south, east and west \( T_e \) directions denoted by N, S, E, W respectively. Here \( \delta x_{PE} \) and \( \delta x_{PW} \) represent the distance between nodes P and E and
nodes P and W respectively.

Finite volume method is used for the discretisation of governing equations on a staggered grid where the scalar component (pressure) is located at the cell center and the velocity components U and V are located at the cell face centers. The finite volume discretised form of Navier-Stokes equation is

\[ \frac{\partial U}{\partial t} + (U,V)U = -\nabla P + \frac{1}{Re^2} \nabla^2 U + \frac{Gr}{Re^2} \xi_j \]

(15)

Where velocity vector, \( \mathbf{U} = U\hat{i} + V\hat{j} + W\hat{k} \) and \( \frac{Gr}{Re^2} \xi_j = 0 \) (term is only applied for Y-direction).

\[ \frac{U^{n+1} - U^n}{\Delta t} = -(U^n \cdot V)U^n - \nabla P^{n+1} + \frac{1}{Re^2} \nabla^2 U^n \]

(16)

For the calculation of intermediate velocity \( U^* \),

\[ \frac{U^* - U^n}{\Delta t} = -(U^n \cdot V)U^n + \frac{1}{Re^2} \nabla^2 U^n \]

(17)

Where \( U^n \) is the velocity field at \( n^{th} \) level.

To determine the pressure field and velocity field, subtract Eq.(17) from Eq.(16).

\[ \frac{U^{n+1} - U^*}{\Delta t} = -\nabla P^{n+1} \]

(18)

From the above equation velocity vector for \( n+1 \) level can be calculated. Values of \( U, V \) and \( P \) at different nodes inside the room can be calculated by this method. These values can be substituted in the non-dimensional energy equation and value of \( \xi \) can be computed which further gives the value of temperature \( T \) at different nodes. In this calculation, time step in the above equations is calculated using the CFL (Courant FriedrichsLewy) method.

### 3.1 Pressure Projection Method

Pressure projection method is an important method for time based incompressible flow problem. When the scalar variable and vector variable is stored at a single node, highly non-uniform pressure field is occurred. Pressure and velocity field can easily decoupled by using this method.

\[ \frac{\partial U}{\partial t} + (U,V)U = -\nabla P + \frac{1}{Re^2} \nabla^2 U + \frac{Gr}{Re^2} \xi_j \]

(15)

Where velocity vector, \( \mathbf{U} = U\hat{i} + V\hat{j} + W\hat{k} \) and \( \frac{Gr}{Re^2} \xi_j = 0 \) (term is only applied for Y-direction).

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(17)

Where \( U^n \) is the velocity field at \( n^{th} \) level.

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\[ \Delta t \leq \frac{\Delta x}{2|\alpha|} \]  

(19)

4. RESULTS AND DISCUSSION

In present work 2D model of room domain of 9m\(^2\) area is used. Firstly, staggered grid method is used for discretising the domain into finite number of (50×50) subdomains. In initial condition, free stream temperature of air inside the room is taken 25°C. All other parameters like conductivity, specific heat, density, domestic viscosity etc. are taken according to room temperature. Study has been conducted for 30 minutes of the clear sunny day. All readings are taken for Ahmedabad city, India. Fiberglass and Rockwool materials are used as an insulation material inside a room. Simulations have been done for 0.11 m, 0.12 m, 0.13 m and 0.14 m insulated wall.

4.1 Variation of AC on-off Cycle for Different Insulations

In present work, Air conditioning system is fixed on west side of wall. When the temperature of room reaches 26.5°C, the AC switches “on” and when the temperature of room reaches 24°C, the AC switches “off”. AC on-off cycle is generated as an output after a long mathematical calculation. In graphs 0 and 1 represents “off” and “on” condition of AC respectively.

The working time of AC increases with the increase in conductivity of insulation material and with the decrease in thickness of insulated wall.

4.2 Temperature Contour Formation

Conduction of heat through the insulated wall is considered one dimensional whereas heat transfer inside the room is considered two dimensional. Solar radiation falling on the outside wall is conducted through the insulated wall thus the wall temperature increased with time. Due to temperature difference between wall and room air, the convection heat transfer process occurs.
Temperature difference causes density difference which results in flow of air inside a room domain by natural convection. AC is installed in the room. Due to the flow of cool air from AC blower side, forced convection process is occurred. Fig.6 shows the temperature contour for both insulation material at different time interval. The region near the AC remains cooler than rest of the room. Thus a vortex is formed in an air circulation.

Rate of heat transfer increases due to higher conductivity of insulation which increases penetration of heat inside the room. Thus, the mean temperature inside the room rises quickly with increase in conductivity of insulation. The load on the AC increases as more heat needs to be dissipated and the AC working (on) time increases.

The rate of Electrical energy saved/day is increased with the thickness of wall. After a certain thickness this value will not be changed much and that thickness is considered as “optimum thickness”. For Fiberglass, Rs31.82 per day can be saved when the wall thickness is 0.130 m. After this point there is a negligible changes in saving in cost of electricity. For Rockwool, Rs31.68 per day can be saved when the wall thickness is 0.130 m.

![Temperature Contour](image1)

**Fig.6** Temperature Contour of (b) Fiberglass and (c) Rockwool at a different time steps.

### 4.3 Calculation of Optimum Thickness

3 star rating AC with a maximum cooling capacity of 1 Ton is used inside the insulated room. 0.983 KW power is used when the compressor is in working condition (Data of carrier 3 Star AC is used). Cost of electricity is taken Rs 5.5 per KWh (unit). Electricity consumed by AC is calculated from the AC on-off outputs. Cost of Electrical Energy Saved/Day at different thickness by using insulation material is shown in fig.7 and fig.8.

![Cost of Electrical Energy Saved/Day](image2)

**Fig.7** Cost of Electrical Energy Saved/Day at different thickness by using Fiberglass insulation.

![Cost of Electrical Energy Saved/Day](image3)

**Fig.8** Cost of Electrical Energy Saved/Day at different thickness by using Rockwool insulation.
5. CONCLUSION

The main objective of the present work is to prepare a mathematical model that identify the energy saving potential of different thermal insulation. The simulations are carried out at different thickness of insulated wall and conductivities to observe the variation of AC time cycle. The amount of electricity consumed and saved per day is calculated on the basis of working time of AC. Energy consumption is increased due to increase in conductivity at the same thickness of insulated wall can be concluded.

REFERENCES


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